Tata Institute of Fundamental Research
Homi Bhabha Road, Mumbai 400005

School of Technology and Computer Science

Instructions for the written test

There are two streams in the School of Technology and Computer Science:

1. Computer Science.
2. Systems Science.

Topics covered in the two streams, as well as some sample questions, are given below.

The question paper will have three parts. Part A is common to both the streams. It will test the general mathematical aptitude of the candidate. There is no prescribed sylabus for Part A. Part B will be oriented towards the topics listed under 'Computer Science' below; and Part C will be oriented towards topics listed under 'Systems Science' below. Only one of Parts B, C, should be attempted. The duration of the written test will be **three hours**. The test will be of **multiple choice type**, with negative marking for incorrect answers. The use of calculators will not be allowed during the test.

Computer Science

1. Discrete Mathematics: Sets and Relations, Combinatorics (Counting) and Elementary Probability Theory, Graph Theory, Propositional and Predicate Logic.
2. Formal Languages, Automata Theory and Computability.
3. Data Structures and Algorithms: Arrays, Lists and Trees, Sorting and Searching, Graph algorithms, Complexity of problems and NP-completeness.
4. Fundamentals of Programming Languages and Compilers: Control structures, Parameter passing mechanisms, Recursion, Parsing and type checking, Memory management.
5. Operating Systems and Concurrency
6. Switching Theory and Digital Circuits
7. Theory of Databases

Sample Questions [Computer Science]

1. A function $f : \{0,1\}^n \to \{0,1\}$ is called *symmetric* if for every $x_1, x_2, \ldots, x_n \in \{0,1\}$ and every permutation σ of $\{1,2,\ldots,n\}$, we have

$$f(x_1, x_2, \ldots, x_n) = f(x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(n)}).$$

   The number of such symmetric functions is:

   (a) 2^{n+1} (b) 2^n (c) $2^{2^n}/n!$ (d) 2^{2^n} (e) n!

2. Let r, s and t be regular expressions. Which of the following is wrong?

   (a) $(r+s)^* = (r^*s^*)^*$ (b) $r(s+t) = rs + rt$
   (c) $(r+s)^* = (s+r)^*$ (d) $(rs+r)^*r = r(sr+r)^*$ (e) All are correct.

3. Consider the following program

```
x:=0; y:=1; z:=1;
while y <= N do
begin
x:=x+1; y:=y+z+2; z:=z+2;
end
```

   Which of the following holds on termination of the program?

   (a) $(x+1)^2 = N$ (b) $x = \sqrt{N}$
   (c) $x^2 = N$ (d) $x^2 \leq N < (x+1)^2$ (e) $x^2 < N \leq (x+1)^2$.

4. The maximum height of a rooted binary tree (all nodes have either two children or none) with N nodes is

   (a) N (b) $\log N$ (c) $(N-1)/2$ (d) $(N^2)/2$ (e) $N(N-1)/2$.

5. If a graph G has n vertices and m edges then the depth first traversal of G can be carried out in time

   (a) $O(n+m)$ (b) $O(nm)$ but not $O(n+m)$
   (c) $O(n^2)$ but not $O(n+m)$ (d) $O(n)$ (e) $O(m)$

Systems Science

1. Engineering Mathematics: Complex Analysis, Linear Algebra, Elementary Numerical Analysis, Basic Optimization Theory and Algorithms, Introduction to Probability Theory and Statistics.

2. Electrical and Computer Sciences: Introduction to Signals and Linear Systems Analysis, Control Systems, Digital Signal Processing, Basic Circuit Theory, Introduction to Digital Communications, Digital Computer Fundamentals, Introduction to Computer Programming.

Sample Questions [Systems Science]

1. The probability density of a random variable is

$$f(x) = ax^2 \exp^{-kx} \ (k > 0, 0 \leq x \leq \infty)$$

Then, the coefficient a equals

(a) $k^3/2$ (b) k^3 (c) k^2 (d) k (e) $2k/\pi$.

2. Discrete sequences $x(n)$ is non-zero for $0 \leq n \leq N_x$ and $y(n)$ for $0 \leq n \leq N_y$. The sequence $z(n)$ is obtained by convolving $x(n)$ and $y(n)$. $z(n)$ assumes nonzero values for $N_1 \leq n \leq N_2$, where N_1 and N_2 can be expressed in terms of N_x and N_y as,

(a) $N_1 = 0; N_2 = MAX(N_x, N_y)$
(b) $N_1 = N_x; N_2 = N_y$
(c) $N_1 = MIN(N_x, N_y); N_2 = N_x + N_y$
(d) $N_1 = 0; N_2 = N_x + N_y$
(e) $N_1 = MIN(N_x, N_y); N_2 = MAX(N_x, N_y)$

3. This is a portion of FORTRAN-77 program for assigning values to a $N \times N$ matrix **A**:

```
DO I=1,N
    DO J=I,N
        A(I,J) = ABS(I-J)+1
    ENDDO
ENDDO
```

What is the matrix **A** called ?

(a) Anti-symmetric (b) Sparse (c) Upper triangular (d) Toeplitz (e) Irregular.

4. $log_b(log_b x)$ equals

(a) $(\ln \ln x - \ln \ln b)/\ln b$
(b) $(\ln x - \ln b)/\ln b$
(c) $(\ln \ln x - \ln \ln b)$
(d) $(\ln x - \ln b)/[(\ln x)(\ln b)]$
(e) None of the Above.

5. The Laplace Transform $G(s)$ of the transfer function of a linear time invariant system is given by

$$G(s) = \frac{1}{(s+a)^2 + b^2}$$

For the system to be stable it is necessary that

(a) $a < 0$ (b) $a \geq 0$ (c) $a = b$ (d) $b = 0$ (e) $a = -b$.